Sustainability at EDMONDS COMMUNITY COLLEGE

Teaching and Learning (T&L) Activity **Submission** Framework

1) Title: Modeling Population Growth

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Dept: Mathematics

Overview: This module, to be used primarily in *Applied Intermediate Algebra* (Math 95), is designed to look at population growth historically and at current and future trends. The students will use real population data for the World and the U.S. to learn about mathematical ideas of Linear models, Exponential models, and Logistic models. As we look at the data and the models for growth, we will discuss the sustainability "big idea" of human population growth and carrying capacity.

2) Introduction and Overview

a) Introductory information about context, rationale, and purpose for this activity. Context would include the type(s) of courses for which this activity would be appropriate.

The physicist and sustainability activist Al Bartlett is quoted as saying, "The greatest shortcoming of the human race is our inability to understand the exponential function." The students in Applied Intermediate Algebra learn about the Exponential function for the first time. They also learn to compare the shapes and patterns of different models (Linear, Exponential, Quadratic, Radical, Rational, and Logistic). In this activity, the students will learn about Exponential functions in the context of population growth. They will compare the Exponential growth model to that of Linear functions (learned previously in the course). Then we will extend the discussion to Logistic models, the most appropriate model we have for population over multiple centuries, and consider the idea of the Earth's carrying capacity.

This module could also be used in *Precalculus I* (Math 141), *Math in Society* (Math 107), or *Business Calculus* (Math 148).

b) Timeframe: how much class-time, and/or how much of the course, this activity will require. Also, where you use this activity in the term: early, middle, late.

I plan to implement this module in the middle of the quarter – during weeks five and six. Exponential Functions are explained in chapter 4 of our current textbook (Lehmann, 2008). I expect it will take about seven class days total to learn exponentials, Exponential functions and Exponential regressions models; then compare the shape and growth rate with those in Linear models and Logistic models. This time estimate includes class discussion on homework, a homework quiz, and in-class activities.

c) The assignment: a brief description of the integrative assignment that includes both the "big idea" in your discipline and the sustainability "big idea;" also any skills or habits of mind you hope the activity will foster.

The mathematical "big idea" is exponential growth and comparing exponential models with linear and logistic models.

The sustainability "big idea" is human population growth and carrying capacity of the Earth.

This integrative assignment will entail several assignments and the “big idea” themes will be woven into multiple class days. It will begin with the start of Exponential functions and models chapter. After looking at a graph of an exponential model for population, we will compare it to a linear graph. After discussing that the graph is of human population numbers, we will get the real population data and use technology to make a scatterplot of the data and do exponential and linear regressions to compare the fit. The assignment also will include some limited reading of articles and/or viewing of YouTube videos on the subject of population growth and exponential growth.

Several days later, we will return to the population growth data to discuss Logistic models, particularly in comparison with Exponential models. At this point, the concept of a carrying capacity will also be emphasized and discussed.

3) **The actual teaching-and-learning activities:**

a) a description of the set-up or preliminaries;

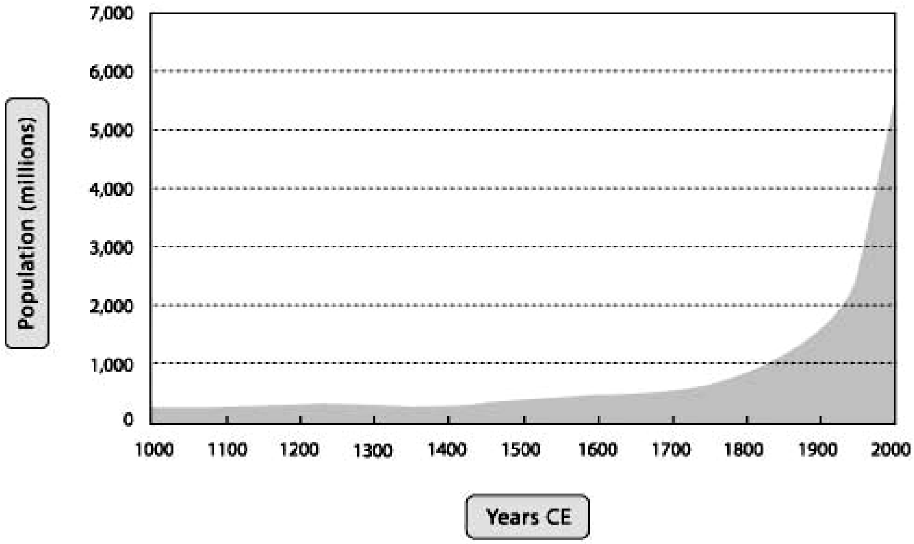
This topic is typically introduced around the fifth week of the quarter after spending about four weeks on linear equations, linear models and functions, and rate of change. The topic of population growth will be the opening problem of the chapter (in Lehmann, Chapter 4) to introduce the concept of a growth pattern that is not linear but one that increases by multiplying instead of adding by a constant. Then, approximately one class day is spent reviewing the rules of working with exponents. Next, the exponential function, $f(x) = ab^x$, is introduced through graphing, and the graphical significance of *a* and *b* is discovered. After taking time to learn how to solve for *a* and *b*, we return to the idea of using Exponential functions to model data. At this point, the following activity (in parts b & c) is given.

b) & c) **the main learning activities; and the assignment task**.

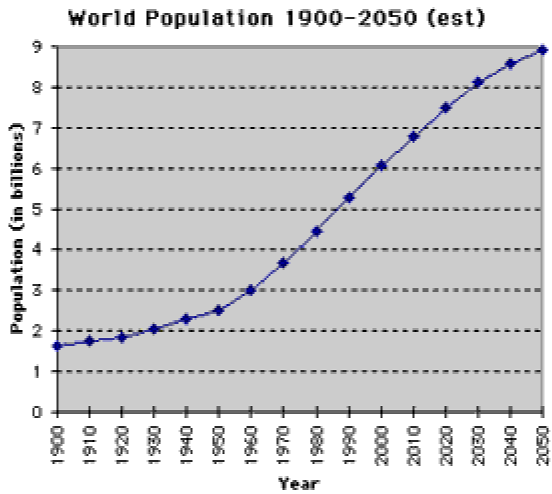
Pages 3-8 include the assignments listed below:

- Several graphs of world population which will be used for the introduction of the Exponential function idea (page 3).
- The “Team Exploration” activity for comparing linear model and exponential model for population growth (page 4)
- The “Team Exploration” activity for modeling population growth with an exponential model including graphing calculator steps (pages 5-6)
- The “Team Exploration” activity for modeling population growth with a logistic model including graphing calculator steps (pages 7-8)

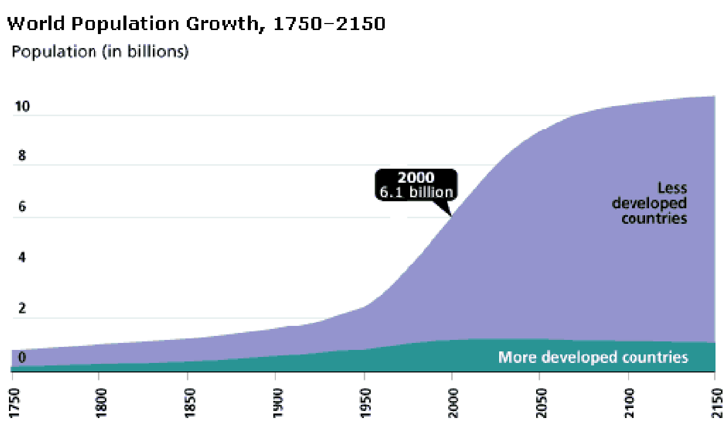
Graphs of World population



http://worldhistoryforusall.sdsu.edu/images/Popn_Graph2.jpg



http://research.biology.arizona.edu/mosquito/willott/323/project/population/worldpop2050.gif



Source: United Nations, *World Population Prospects, The 1998 Revision*; and estimates by the Population Reference Bureau.

http://members.cox.net/slsturgi3/population_growth.gif

Math 95
Group Exploration- 4.5 (a)

Your Name ____________________________

Group Member(s) ____________________________

Exponential Models vs. Linear Models

OBJECTIVE: To compare a linear model with an exponential model

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In 1950, world population was 2.5 billion. In 1987, it was 5.0 billion. (Source: U.S. Census Bureau)

1. First, assume that world population is growing exponentially. Let $E(t)$ be the world's population (in billions) at t years since 1950. Find an equation of E.

2. Now assume that world population is growing linearly. Let $L(t)$ be the world's population (in billions) at t years since 1950. Find an equation of L.

3. Use your equations of E and L to make two predictions of the world's population for each of the following years.
   a. 2010
   b. 2050
   c. 2150

4. Use the window settings given below to graph E and L. Compare. Sketch a graph of each, labeling the graphs.

```
WINDOW
Xmin=0
Xmax=200
Xscl=50
Ymin=0
Ymax=100
Yscl=10
Xres=1
```

5. Will there be much difference in the world's population if it grows exponentially or linearly in the short run? In the long run? Explain. Use the back side if you need additional room.

Math 95 Your Name ____________________________
Group Exploration- 4.5 (b)
Group Member(s) ____________________________

Finding the Equation of an Exponential Model Using Technology

OBJECTIVE: To introduce using a graphing calculator to perform an Exponential Regression to find an equation of an exponential function.

The population of the world grew tremendously over the 20th Century. Data from 1900 to 2005 is shown in the table below.

Year	World Population (in billions)
1900	1.6
1920	1.8
1930	2.0
1940	2.2
1950	2.4
1960	2.9
1970	3.7
1980	4.4
1990	5.1
2005	6.5

Source: Vaughn Aubuchon Summary

Let $P=f(t)$ be the Population at t years since 1900.

1. Use a graphing calculator to draw a scattergram of the data. Sketch the picture.

2. Is it better to use a linear or an exponential function to model the data? Explain.

3. Using your graphing calculator (see below), find an equation for $f(t)$.

Steps for finding an exponential regression equation and pasting it as y1 (TI-83/84):
1. **STAT**;
2. using right arrow, go to **CALC**
3. go to **#0: ExpReg;** press **ENTER**
4. to type in "Y1" to paste, select **VARS**
5. using right arrow, go to **Y-VARS**
6. Press **ENTER** to select **1:Function**
7. Press **ENTER** to select **1:Y1** (now you should see "ExpReg Y1" on your screen.
8. Press **ENTER**

9. Record equation (either from Home screen or **Y=** Screen) using correct notation and labels.

Note: If you get "ERR:OVERFLOW" on the calculator screen when attempting to compute the exponential regression, that means you forgot to align your data (you entered "1980, 1982,…" instead of "0, 2, …" for your L1 list).

Group Exploration- 4.5 (b) Continued

II. The population of the U.S. also has grown significantly over the last half of the 20th Century. Data from 1950 to 1990 is shown in the table below.

Year	US Population (in thousands)
1950	152
1955	166
1960	180
1965	194
1970	205
1975	216
1980	228
1985	238
1990	250

Source: U.S. Census Bureau

Note: Values for the middle of each given year.

Let $P=f(t)$ be the Population of the U.S. at t year since 1950.

1. Use a graphing calculator to draw a scattergram of the data. Sketch a picture.

2. Is it better to use a linear or an exponential function to model the data? Explain.

3. Using your calculator, find the regression model for the data. Use proper notation and labels.

Math 95 Your Name ____________________________
Group Exploration- 4.5 (c)

Group Member(s) ____________________________

More Population Modeling - Finding the Equation of an Logistic Model Using Technology

OBJECTIVE: To introduce a Logistic Regression to find an equation of a logistic function to model human population growth. We will also introduce the concept of an asymptote (carrying capacity).

The population of the world grew tremendously over the 20th Century. Data from 1900 to 2005 is shown in the table below.

Year	World Population (in billions)
1900	1.6
1920	1.8
1930	2.0
1940	2.2
1950	2.4
1960	2.9
1970	3.7
1980	4.4
1990	5.1
2005	6.5

Source: Vaughn Aubuchon Summary

Recall in Exploration 4.5(b) we used an exponential function to model the data. However, increasing exponential models increase without bound.

1. Do you think the human population of the Earth will continue to grow exponentially without bound? Explain why or why not.

Many people feel that there is an upper bound, a "carrying capacity" of the Earth. Mathematically, this is displayed as a *horizontal asymptote*. There is another model that changes exponentially for a time, but then levels off at an upper (or lower) limit. It is called a *Logistic Model*.

2. As you did in Activity 4.5 (b), enter the data into your calculator and draw a scattergram of the data. Sketch the picture.

3. Let $P=f(t)$ be the Population at t years since 1900. We will now use the Logistic model which is given by

$$f(t)=\frac{c}{1+ae^{-bt}}$$

where a, b, and c are constants; and c is the upper bound (horizontal asymptote).

However, the calculator is particular about the data used for a Logistic model, so we need to adjust it so that it starts with a lower bound near $y=0$. Thus, we will subtract 1 from each y-data point. To do this, go to your L2 list, and with the curser on the "L2" type in "L2-1" followed by ENTER. All the data in L2 should now be 1 less than the given data. (Note: you can do this adjustment by hand if you prefer.)

4. Now follow the steps below to find a logistic equation for $f(t)$.

Steps for finding an logistic regression equation and pasting it as y1 (TI-83/84):

i. **STAT**;
ii. using right arrow, go to **CALC**
iii. go to **#B: Logistic;** press **ENTER**
iv. to type in "Y1" to paste, select **VARS**
v. using right arrow, go to **Y-VARS**
vi. Press **ENTER** to select **1:Function**
vii. Press **ENTER** to select **1:Y1** (now you should see "Logistic Y1" on your screen.
viii. Press **ENTER**

5. Before recording the equation, we need to recall we subtracted "1" from each y-value. Thus, we need to add "1" to the function. Now record the equation (either from Home screen or **Y=** Screen) and follow it with "+1". Don't forget to use correct notation and labels.

Hints: your recorded equation should be in the form: $f(x)=\frac{c}{1+ae^{-bx}}+1$. Also, make this change in your calculator under Y1.

6. Enlarge your Window on the calculator so that your Xmax=200 and your Ymax=20.
   a. Using the model from (5), predict the population in 2050.

   b. Using the model from (5), estimate the "carrying capacity" of the Earth.

   c. Do you think the actual "carrying capacity" of the Earth is higher or lower than that predicted in (b)? Explain.

4) **Optional Assessment elements --- if not already described in (3), describe how you will assess student learning in this assignment.**

   Student learning of the mathematical concepts will be assessed with the team activities in (3), as part of a weekly quiz, and as part a chapter project or test.

5) **Resources, if needed: Community partners, websites, written material, videos/film, etc. Limit these to resources that support or extend <u>this activity</u>, not resources related to your entire course.**

   Several websites will be utilized for data, videos, and materials. Some of them are given in the References list (page 10).

6) **Optional Teacher notes: Any reflections or cautions, or special suggestions or observations related to the student learning you have observed.**

   None at this time.

Resources:

Textbook:

1. Lehmann, Jay. *Intermediate Algebra; Functions & Authentic Applications*. 3rd Ed. Pearson Prentice Hall. New Jersey. 2008.

Web sites:

1. Consumption and Growth 101 - exponential growth calculator and short tutorial on exponential growth: www.consumptiongrowth101.com
2. US Census Bureau: http://www.census.gov/ipc/www/idb/
3. YouTube Video: Are humans smarter than yeast? - video on exponential growth.
4. Al Bartlett's home page: http://www.albartlett.org/index.html
5. Making the Link: Population, Health, and the Environment: http://www.prb.org/pdf/MakingTheLinkPHE-be.pdf
6. Population Reference Bureau: http://www.prb.org/Publications/Datasheets.aspx